Comment on "Spin manipulation of 1.94 GeV/c polarized protons stored in the COSY cooler synchrotron"

M. Bai, W. W. MacKay, and T. Roser

Brookhaven National Laboratory, Upton, New York 11720, USA

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We show that the formula of the strength of the spin resonance driven by a rf dipole in the commented paper is incorrect. A derivation of the resonance strength due to direct spin rotation from a rf dipole is shown. The result of a numerical simulation to verify our derivation is also presented.

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I. INTRODUCTION

In [1], the authors describe the experiment of using a rf dipole to achieve full spin flip of the 1.94 GeV/c polarized proton beam in the COSY cooler ring. The authors stated that the strength of the spin resonance induced by the rf dipole is given by

$$\epsilon = \frac{1 + G\gamma}{\sqrt{2}\pi} \frac{\int B_{\mathrm{rms}} dl}{B\rho}, \tag{1}$$

where γ is the Lorentz factor, $G = \frac{g-2}{2}$ is the anomalous g factor, $\int B_{\mathrm{rms}} dl$ is the rms value of the integrated oscillating rf dipole field, and $B\rho = \frac{p}{q}$ is the beam rigidity, i.e., momentum per charge. However, Eq. (1) is incorrect by a factor of 2 in the denominator. The correct expression of the strength of a spin resonance excited by the spin rotation directly produced by a rf dipole is given by [2]

$$\epsilon = \frac{1 + G\gamma}{4\pi} \frac{\int B_{\mathrm{osc}} dl}{B\rho} = \frac{1 + G\gamma}{2\sqrt{2}\pi} \frac{\int B_{\mathrm{rms}} dl}{B\rho}, \tag{2}$$

where $B_{\mathrm{osc}} = \sqrt{2} B_{\mathrm{rms}}$ is the amplitude of the oscillating dipole field. The rf dipole also generates a coherent orbital oscillation which causes an additional contribution to the spin resonance strength. The phase and strength of this additional contribution strongly depend on the lattice parameters. A derivation of Eq. (2) together with numerical simulation will be presented in the following sections.

II. DERIVATION

In a planar accelerator, the precession of the spin vector $\vec{S}$ is governed by the Thomas–Bargmann-Michel-Telegdi (Thomas-BMT) equation [3]

$$\frac{d\vec{S}}{dt} = \vec{S} \times \frac{e}{m\gamma}[(1 + G\gamma)\vec{B}_\perp + (1 + G)\vec{B}_\parallel], \tag{3}$$

where $\vec{B}_\perp$ and $\vec{B}_\parallel$ are the magnetic field components transverse and parallel to the direction of the particle, respectively.

In a simple model consisting of a perfect machine with a rf dipole as the only perturbing field of the spin precession, one can express the spin precession equation of Eq. (3) in the frame which rotates with the particle's orbital revolution as

$$\frac{d\vec{S}}{d\theta} = \vec{S} \times \left[G\gamma\hat{y} + (1 + G\gamma)\frac{B_{\mathrm{osc}} L(\theta)}{B\rho}\cos(Q_{\mathrm{osc}}\theta + \chi)\hat{x}\right], \tag{4}$$

where $d\theta = \frac{v}{\rho} dt$, v is the speed of the particle, θ is the bending angle in the main dipoles, and $B_{\mathrm{osc}} L(\theta)$ is the amplitude of the integrated strength of the rf dipole oscillating field

$$B_{\mathrm{osc}} L(\theta) = B_{\mathrm{osc}} L \delta(\theta - \theta_{\mathrm{osc}}) \tag{5}$$

where

$$\int_0^{2\pi} \delta(\theta - \theta_{\mathrm{osc}}) d\theta = 1. \tag{6}$$

Here θ_{osc} is the azimuthal angle of the location of the rf dipole, Q_{osc} is the oscillating frequency of the rf dipole field in units of the particle's revolution frequency, and χ is the initial phase of the oscillating field. The corresponding spinor equation is

$$\frac{d\psi(\theta)}{d\theta} = -\frac{i}{2}\left[G\gamma\sigma_3 + (1 + G\gamma)\frac{B_{\mathrm{osc}} L(\theta)}{B\rho} \times \frac{e^{i(Q_{\mathrm{osc}}\theta+\chi)\sigma_3} + e^{-i(Q_{\mathrm{osc}}\theta+\chi)\sigma_3}}{2}\sigma_1\right]\psi(\theta). \tag{7}$$

To calculate the amount of kick the spin vector gets from the rf dipole in one orbital turn, one can transform Eq. (7) into the frame which rotates at the same frequency as the rf dipole oscillating frequency; namely, let

$$\Psi(\theta) = e^{(i/2)(Q_{\mathrm{osc}}\theta+\chi)\sigma_3}\psi(\theta). \tag{8}$$

Equation (7) then becomes

$$\frac{d\Psi(\theta)}{d\theta} = -\frac{i}{2}(G\gamma - Q_{\mathrm{osc}})\sigma_3\Psi(\theta) - \frac{i}{2}\frac{1 + G\gamma}{2} \times \frac{B_{\mathrm{osc}} L(\theta)}{B\rho} e^{(i/2)(Q_{\mathrm{osc}}\theta+\chi)\sigma_3}(e^{i(Q_{\mathrm{osc}}\theta+\chi)\sigma_3} + e^{-i(Q_{\mathrm{osc}}\theta+\chi)\sigma_3})\sigma_1 e^{-(i/2)(Q_{\mathrm{osc}}\theta+\chi)\sigma_3}\Psi(\theta). \tag{9}$$

This then leads to

$$\frac{d\Psi(\theta)}{d\theta} = -\frac{i}{2}(G\gamma - Q_{\rm osc})\sigma_3\Psi(\theta) - \frac{i}{2}\frac{1+G\gamma}{2}\frac{B_{\rm osc}L(\theta)}{B\rho} \times (e^{2i(Q_{\rm osc}\theta+\chi)\sigma_3}+1)\sigma_1\Psi(\theta). \quad (10)$$

When $G\gamma = Q_{\rm osc}$, Eq. (10) becomes

$$\frac{d\Psi(\theta)}{d\theta} = -\frac{i}{2}\frac{1+G\gamma}{2}\frac{B_{\rm osc}L(\theta)}{B\rho}e^{2i(Q_{\rm osc}\theta+\chi)\sigma_3}\sigma_1\Psi(\theta) - \frac{i}{2}\frac{1+G\gamma}{2}\frac{B_{\rm osc}L(\theta)}{B\rho}\sigma_1\Psi(\theta). \quad (11)$$

Since the contribution of the first term in Eq. (11) averages to zero, the spinor motion in the rotating frame simplifies to

$$\Psi(\theta + 2\pi) = e^{-(i/2)(1+G\gamma/2)(B_{\rm osc}L/B\rho)\sigma_1}\Psi(\theta), \quad (12)$$

and the one turn transfer matrix M of the spinor is

$$M = \begin{pmatrix} \cos(\frac{1+G\gamma}{2}\frac{B_{\rm osc}L}{2B\rho}) & -i\sin(\frac{1+G\gamma}{2}\frac{B_{\rm osc}L}{2B\rho}) \\ -i\sin(\frac{1+G\gamma}{2}\frac{B_{\rm osc}L}{2B\rho}) & \cos(\frac{1+G\gamma}{2}\frac{B_{\rm osc}L}{2B\rho}) \end{pmatrix}. \quad (13)$$

The amount of spin rotation angle ϕ directly from the rf dipole over one revolution is determined by

$$\cos\left(\frac{\phi}{2}\right) = \frac{1}{2}\,{\rm tr}(M). \quad (14)$$

Equation (14) then gives

$$\phi = \frac{1+G\gamma}{2}\frac{B_{\rm osc}L}{B\rho}. \quad (15)$$

Hence, the spin resonance strength ϵ is

$$\epsilon = \frac{\phi}{2\pi} = \frac{1+G\gamma}{4\pi}\frac{B_{\rm osc}L}{B\rho}, \quad (16)$$

which agrees with Eq. (2).

III. NUMERICAL SIMULATION

A numerical simulation was done to verify Eq. (2). The simulation was a simple spin rotation around a circular ring with $G\gamma = 46.28$. In this model, the spin precesses around the vertical axis by $G\gamma = 46.28$ times every orbital revolution. At the end of each orbital turn, a rf dipole with transverse horizontal magnetic field is met, and the spin vector then gets precessed around the horizontal axis by an angle of $(1 + G\gamma)\frac{B_kL}{B\rho}$. Here, $B_kL = B_{\rm osc}L\cos(2\pi k Q_{\rm osc})$ is the rf dipole field on the kth turn.

In this simulation, the rf dipole strength is set to $B_{\rm osc}L = 0.005$ Tm. First, the rf dipole tune was kept fixed for 1000 turns at $Q_{\rm osc} = 0.23$. It is then swept linearly for 5000 turns from 0.23 to 0.33. At the end of the rf dipole tune sweep, the spin was tracked for another 1000 turns. Figure 1 shows the vertical component of spin S_y for a single proton as a function of orbital turns. A spin resonance is observed at $G\gamma - {\rm int}[G\gamma] = Q_{\rm osc}$, and it behaves as a regular isolated spin resonance. Hence, the strength of this rf induced resonance can then be calculated from the Froissart-Stora formula [4]

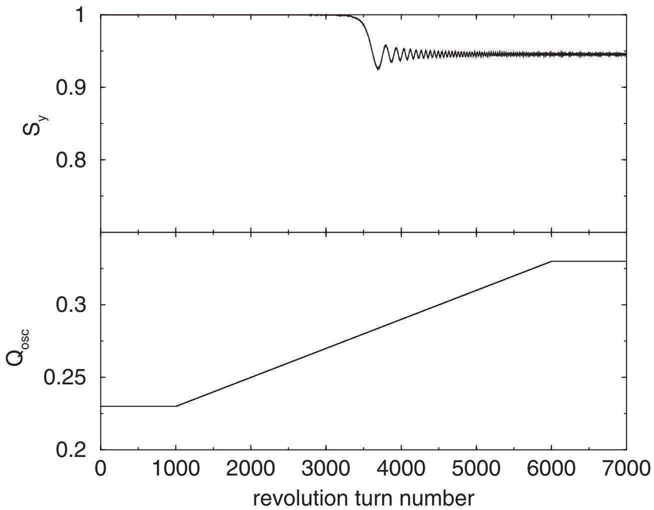

FIG. 1. Numerical simulation of spin vector of a particle crossing a rf dipole driven oscillation. The top curve shows the vertical component of the spin vector as a function of orbital turns. The bottom curve shows the rf dipole oscillating frequency versus the orbital turn number. A spin resonance is seen at $G\gamma - {\rm int}[G\gamma] = Q_{\rm osc}$, with $Q_{\rm osc} = 0.28$.

$$\frac{P_f}{P_i} = 2\exp\left(-\frac{\pi|\epsilon|^2}{2\alpha}\right) - 1, \quad (17)$$

and $\alpha = \frac{dQ_{\rm osc}}{d\theta}$ is the resonance crossing rate. For this simulation, the resonance crossing rate is

$$\alpha = \frac{dQ_{\rm osc}}{d\theta} = \frac{0.1}{5000 \times 2\pi} = 3.18 \times 10^{-6}. \quad (18)$$

By taking the ratio of the averages of the first and last 1000 turns

$$\frac{\langle P_f\rangle}{\langle P_i\rangle} = 0.9459, \quad (19)$$

the spin resonance strength is then calculated as

$$\epsilon = 0.000\,236. \quad (20)$$

This is in good agreement with Eq. (2), which gives $\epsilon = 0.000\,233$.

In an accelerator, the rf dipole with horizontal magnetic field not only can induce a spin resonance but also can excite a vertical coherent betatron oscillation [5]. The size of the coherent oscillation is proportional to the inverse of $|Q_y - Q_{\rm osc}|$, with Q_y being the vertical betatron tune. This coherent oscillation causes additional perturbations on the spin motion due to the quadrupole fields. This effect then interferes with the rf dipole driven spin resonance [6]. Depending on the lattice parameters, the perturbation on the spin motion from the vertical coherent oscillation can be either in phase or out of phase with the rf dipole driven spin resonance. This could be the reason for the discrepancy between the COSY experimental data and Eq. (2). The details of the contributions of the additional spin rotation from the rf dipole driven vertical coherent oscillation will be presented in our future publications.

IV. CONCLUSION

A rf dipole or solenoid with oscillating magnetic field can be used to induce a spin resonance at the location of its oscillating frequency. This induced spin resonance behaves the same as the regular isolated spin resonance. A derivation of the rf dipole driven resonance strength is shown in the Sec. II, and the strength ϵ is given by Eq. (2).

Unlike the rf solenoid which only perturbs the spin motion, the rf dipole also excites a vertical coherent betatron oscillation. This coherent oscillation introduces additional perturbations on the spin motion due to the quadrupole fields the particle experiences along its trajectory. This effect can either cancel or enhance the rf dipole driven resonances depending on the lattice parameters.

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