Parameters of Compton x-ray beams: Total yield and pulse duration

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Closed analytical formulas describing the parameters of x rays, generated in Compton backscattering of laser pulse and electron beam, such as total yield and temporal duration of the x-ray pulse are presented. Despite the fact that maximal yield is attainable at the head-on collisions and the analytical expressions for this specific case were derived, for some technological reasons in the Compton sources, the head-on collision is not acceptable. In ''laser wire'' monitors the laser beam crosses the electron bunch at the right angle. A model comprising the ellipsoidal laser and electron bunches with tri-Gaussian density distribution is considered. For this model, the total yield of x-ray quanta and temporal duration is derived as functions of the (arbitrary) crossing angle and geometric dimensions of both the bunch and the pulse. As is shown, the yield is maximal for the head-on collision. With increase in the collision angle, the yield diminishes with rate determined by dimensions of the bunches. Duration of the x-ray pulse behaves in a similar way; minimal duration is attained near the back-on collisions.

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I. INTRODUCTION

Backscattering of a high intensity laser pulse with a bunch of relativistic electrons is a promising method for producing bright short x-ray radiation pulses within a compact facility—Compton sources [1,2]. Additionally, this process is applied for the nondestructive monitoring of electron beams—in the ''laser wire'' monitors [3]. For these and similar applications, major figures of merit are the total yield (number of x-ray quanta emitted per a collision) and the temporal duration if ultrashort x-ray pulses are required.

Despite the fact that maximal yield is attainable at the head-on collisions and the analytical expressions for this specific case were derived, for some technological reasons in the Compton sources, head-on collisions are not acceptable. In the monitors the laser wire penetrates the electron beam side-on, at a right angle.

Based on a model of ellipsoidal laser and electron bunches colliding at an arbitrary angle, analytical formulas have been derived. The formulas in limiting cases coincide with the known formulas from published papers.

The paper is organized as follows. In Sec. II a setup of the model is described as well as a method of analysis: the corpuscular approach based on an invariant cross section for the Compton scattering. A closed analytical formula describing the total x-ray yield for arbitrary collision angle, electron energy, and geometry is derived. Also there are presented reduced expressions for the limiting cases. In Sec. III, described are the temporal duration of the x-ray pulse derived with similar techniques as the yield. The paper is concluded with a survey of the main results and discussion on applicability of them.

II. YIELD OF SCATTERED QUANTA

A. Invariant form of cross section

The number of collisions between particles of two different species (in a rest frame of one of the species) is proportional to the density of each species, the interaction cross section, and a relative velocity of particles (see [4]):

$$\dot{n} = \sigma n_e n_1 v_{1,2}, \tag{1}$$

where $\dot{n}$ is the number of collisions per unit volume per unit time interval, $n_{1,2}$ the density of the first and second specie, respectively, v_{rel} their relative velocity, and σ the scattering cross section.

The total yield of secondary particles is equal to the integral of $\dot{n}$ over the volume and time interval of the beam-to-pulse collision:

$$N = \sigma v_{\mathrm{rel}} \int_V \int_{\Delta t} n_e(\mathbf{r}, t) n_l(\mathbf{r}, t) \mathrm{d}^3\mathbf{r}\mathrm{d}t. \tag{2}$$

It can easily be seen that Eq. (2) is reduced to the well-known limiting cases of the projectile-to-target collision or counterpropagating bunches.

In a reference coordinate frame with the both bunches moving at relativistic velocities, and with particles at rest in the rest coordinates (no particle motion within the bunch, trajectories parallel), the expression for the number of pair collisions reads (as was shown by Pauli, see, e.g., [4])

$$N = \sigma S \int_V \int_{\Delta t} n_e(\mathbf{r}, t) n_l(\mathbf{r}, t) \mathrm{d}^3\mathbf{r}\mathrm{d}t, \tag{3}$$

where

$$S = \sqrt{(\mathbf{v}_1 - \mathbf{v}_2)^2 - \frac{[\mathbf{v}_1\mathbf{v}_2]^2}{c^2}},$$

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$\mathbf{v}_{1,2}$ are particles (and bunch) velocities in the laboratory coordinate frame. A similar approach was used in [5–7].

B. Gaussian bunches colliding at arbitrary angle

Consider the crossing of the photon bunch (laser pulse) and the electron bunch in the laboratory coordinate frame x, y, z. (We suggest a conventional ''accelerator'' coordinate frame depicted in Fig. 1.)

We suggest the electron bunch propagating along y axis with velocity $v_e = v_y$ in a coordinate system (CS) (x, y, z); the photon pulse contrapropagates along the y' axis in CS (x', y', z'). The photon CS is turned with respect to the electron's by angle φ around the z axis (coincident with the z' axis). The relations between the electron CS and the photon CS read

$$x' = x\cos\varphi + y\sin\varphi; \tag{4a}$$

$$y' = -x\sin\varphi + y\cos\varphi; \tag{4b}$$

$$z' = z. \tag{4c}$$

We suggest the tri-Gaussian distribution of both the electrons and the photons within bunches:

$$n_e = \frac{N_e}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z}\exp\frac{1}{2}\left[-\frac{x^2}{\sigma_x^2} - \frac{(y-\beta T)^2}{\sigma_y^2} - \frac{z^2}{\sigma_z^2}\right],$$
$$n_l = \frac{N_l}{(2\pi)^{3/2}\sigma'_x\sigma'_y\sigma'_z}\exp\frac{1}{2}\left[-\frac{x'^2}{\sigma_x'^2} - \frac{(y'+T)^2}{\sigma_y'^2} - \frac{z'^2}{\sigma_z'^2}\right], \tag{5}$$

where $N_{e,l}$ is the total number of electrons and photons in bunches, respectively; $\sigma_{x,y,z}$ the dimensions of electron bunch; $\sigma'_{x,y,z}$ the dimensions of photon pulse; $\beta = v_e/c$ the relative electron speed (c the speed of light in vacuum); $T = ct$ the time variables (in length units).

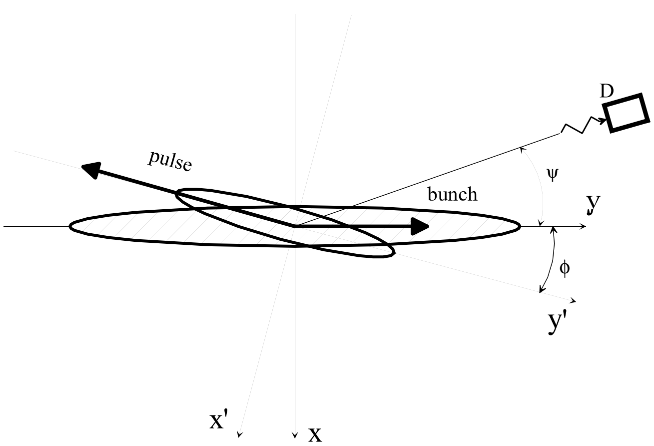

FIG. 1. The electron and photon coordinate frames: z coincides with z' and directed upward from the (x, y) plane. The electron bunch is dashed, D is a detector of x rays.

In the model, the transversal dimensions of the laser pulse are suggested constant since the laser pulse considered is short compared with the Rayleigh range of an optical resonator. The longitudinal density profile of such short laser pulses obeys the Gaussian law; see [8].

This model of the colliding bunches enables us to get results in a closed analytical form without significant loss of generality (see [9]).

Making use of the above-described bunch model, we derive the yield for a general case of collision at an arbitrary angle φ. Integration of (3) with an account for the Gaussian bunch density shapes and $v_{\mathrm{ph}} = c$ finally results in

$$Y = \mathcal{G}\iiiint \exp\left[-\frac{\mathcal{F}(x, y, z, T)}{2}\right]\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}T, \tag{6}$$

where

$$\mathcal{G} = \frac{N_e N_l \sigma(1+\beta\cos\varphi)}{(2\pi)^3\sigma_x\sigma_y\sigma_z\sigma'_x\sigma'_y\sigma'_z}.$$

Here $\mathcal{F}(x, y, z, T)$ is a quadratic form of type

$$\mathcal{F}(x, y, z, T) = \sum_{i=1}^{4}\sum_{j=1}^{4} a_{ij}\xi_i\xi_j. \tag{7}$$

Here $\xi = (x, y, z, T)$, a $a_{ij} = a_{ij}(\varphi, \sigma_x, \sigma_y, \sigma_z, \sigma'_x, \sigma'_y, \sigma'_z)$ is a symmetric matrix consisting of the terms

$$a_{11} = \frac{1}{\sigma_x^2} + \frac{\cos^2\varphi}{\sigma_x'^2} + \frac{\sin^2\varphi}{\sigma_y'^2}; \qquad a_{14} = -\frac{\sin\varphi}{\sigma_y'^2};$$

$$a_{12} = \left(\frac{1}{\sigma_x'^2} - \frac{1}{\sigma_y'^2}\right)\cos\varphi\sin\varphi; \qquad a_{24} = \frac{\cos\varphi}{\sigma_y'^2} - \frac{\beta}{\sigma_y^2};$$

$$a_{22} = \frac{1}{\sigma_y^2} + \frac{\sin^2\varphi}{\sigma_x'^2} + \frac{\cos^2\varphi}{\sigma_y'^2}; \qquad a_{44} = \frac{\beta^2}{\sigma_y^2} + \frac{1}{\sigma_y'^2};$$

$$a_{33} = \frac{1}{\sigma_z^2} + \frac{1}{\sigma_z'^2}; \qquad a_{i3} = 0.$$

After a reduction of the matrix to the diagonal form, Eq. (7) reads

$$\mathcal{F}(x, y, z, T) = \sum_{i=1}^{4}\mathcal{A}_{ii}\eta_i^2, \tag{8}$$

where elements η_i are defined through x, y, z, T as

$$\eta_1 = x + \frac{a_{12}}{a_{11}}y + \frac{a_{14}}{a_{11}}T;$$

$$\eta_2 = y + \frac{a_{11}a_{24} - a_{12}a_{14}}{a_{11}a_{22} - a_{12}^2}T; \qquad \eta_3 = z; \qquad \eta_4 = T,$$

and $\mathcal{A}_{ij}$ are expressed through a_{ij} as

$$\mathcal{A}_{11} = a_{11}; \qquad \mathcal{A}_{22} = \frac{a_{11}a_{22} - a_{12}^2}{a_{11}}; \qquad \mathcal{A}_{33} = a_{33};$$

$$\mathcal{A}_{44} = \frac{(a_{11}a_{44} - a_{14}^2)}{a_{11}} - \frac{(a_{11}a_{24} - a_{12}a_{14})^2}{a_{11}(a_{11}a_{22} - a_{12}^2)}. \tag{9}$$

Since Jacobian for the transition from the variables x, y, z, T onto $\eta_1, \eta_2, \eta_3, \eta_4$ is equal to unity, Eq. (6) can read as

$$Y = \mathcal{G} \iiiint \exp\left(-\frac{\mathcal{A}_{ii}\eta_i^2}{2}\right) d\eta_1 d\eta_2 d\eta_3 d\eta_4, \tag{10}$$

with summation over repeating indexes being presumed.

The integration of Eq. (10) over spatial infinite limits and a finite temporal interval $-L \leq \eta_4 \leq L$ yields

$$Y(\varphi) = \frac{N_e N_{\rm ph}\sigma}{2\pi\sqrt{\sigma_z^2 + \sigma_z'^2}} \times \frac{\mathrm{erf}(u)}{\sqrt{\sigma_x'^2 + (\sigma_y^2 + \sigma_y'^2\beta^2)(\frac{\sin\varphi}{1+\beta\cos\varphi})^2 + \sigma_x^2(\frac{\beta+\cos\varphi}{1+\beta\cos\varphi})^2}}$$

$$(u = \sqrt{\mathcal{A}_{44}/2}L). \tag{11}$$

As can be seen from (11), for a sufficiently general model, dependence of the total yield of x rays upon the bunches geometry, the velocity of electrons, and the crossing angle is rather intricate.

C. Yield for ultrarelativistic electrons

The derived dependence will be much simpler for the most interesting applications case of ultrarelativistic electrons, $1 - \beta \ll 1$ (or $\gamma \gg 1$). Assuming $\beta = 1$ and $L \to \infty$ in (11), we get

$$Y(\varphi) = \frac{N_e N_{\rm ph}\sigma}{2\pi\sqrt{\sigma_z^2 + \sigma_z'^2}} \frac{1}{\sqrt{\sigma_x^2 + \sigma_x'^2 + (\sigma_y^2 + \sigma_y'^2)\tan^2\frac{\varphi}{2}}}. \tag{12}$$

This formula differs from that presented in [10] at $\varphi > 0$ coinciding with it for the head-on collision.

To illustrate the response of the yield $Y(\varphi)$ upon variation of parameters σ_x, σ_x', σ_y, σ_y', φ, let us consider a quantity

$$\frac{Y(\varphi)}{Y(0)} = \left[1 + \left(\frac{\tan\varphi/2}{\delta}\right)^2\right]^{-1/2}, \tag{13}$$

where $\delta^2 \equiv (\sigma_x^2 + \sigma_x'^2)/(\sigma_y^2 + \sigma_y'^2)$ characterizes the ratio of the transversal dimensions of bunches to the longitudinal ones.

Dependencies of the relative yield upon the crossing angle φ at different values of δ are plotted in Fig. 2.

As follows from the plot, the response of the yield upon an increase in the crossing angle is sharper for the longer bunches, $\delta \to 0$. For the case of "ball-like" bunches, $\delta = 1$, dependence (13) becomes simply

$$Y(\varphi) = Y(0)\cos\varphi/2.$$

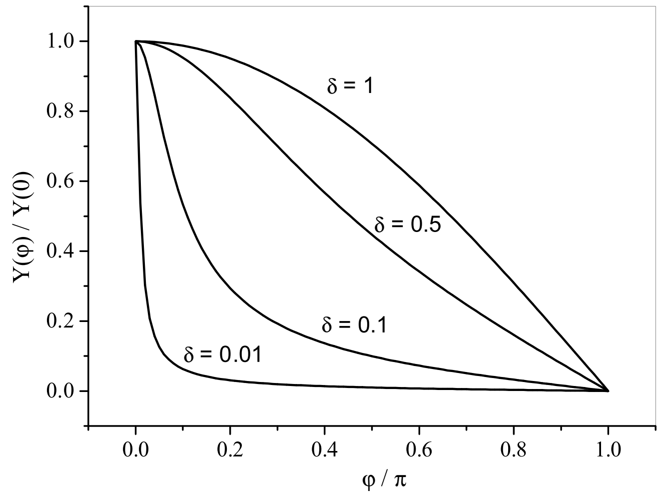

FIG. 2. Dependencies of $Y(\varphi)/Y(0)$ on φ at various δ.

D. Back-on collision $\pi - \varphi \ll 1$

Equation (12) produces zero yield at $\varphi = \pi$. This nonphysical result is a consequence of the assumption $\beta = 1$: there is no collision at equal velocities of the bunches.

On the other hand, it is clear that the yield for back-on collision must be equal to the head-on one since these two cases are not distinguishable in the rest frame of the electron bunch.

To derive the yield at crossing angles close to π, we should start from the general expression (11). Additionally, finite length L of the interaction section should be taken into account.

Substituting $\varphi = \pi$ into the general yield (11), we get

$$Y(\pi) = \frac{N_e N_{\rm ph}\sigma}{2\pi\sqrt{(\sigma_z^2 + \sigma_z'^2)(\sigma_x^2 + \sigma_x'^2)}} \mathrm{erf}(u), \tag{14}$$

with u at $\gamma \gg 1$ being

$$u \approx L[2\gamma^2\sqrt{2(\sigma_y^2 + \sigma_y'^2)}]^{-1}.$$

As has been anticipated, the back-on yield is equal to the head-on if the length of interaction is sufficiently long, $L \to \infty$.

Behavior of the yield in the vicinity of $\varphi = 0$ and $\varphi = \pi$ is entirely different. We recall the yield at $\varphi \ll 1$, as it can be derived from (12) or (13), being

$$Y(\varphi \ll 1) \approx Y(0)\left(1 - \frac{\varphi^2}{8\delta^2}\right). \tag{15}$$

At crossing angles close to π (and $\gamma \gg 1$), the yield can read

$$Y(\pi - \varphi \ll 1) \approx Y(0)\left[1 - \frac{4\gamma^4}{\delta^2}(\pi - \varphi)^2\right], \qquad (16)$$

with natural condition of validity $\pi - \varphi < \delta/(2\gamma^2)$.

Comparing (16) with (15), a fast raise of yield when $\varphi \to \pi$ takes place, $32\gamma^2$ times faster than at $\varphi \to 0$ (it is shown in Fig. 3).

E. Yield in a practical system

Dependence of the total yield of x-ray quanta upon the crossing angle evaluated with (11) for a practical case of the bunches parameters from Table I is presented in Fig. 3.

As can be seen from the picture, the yield reaches its maximal magnitude for the head-on collision ($\varphi = 0$). It then decreases rather quickly with an increase in the angle. At the back-on collision, $\varphi \approx \pi$, the yield sharply reaches its maximum (the infinitely long interacting length has been suggested: $L \to \infty$).

F. Laser wire yield

In the case of laser wire, when a crossing angle is right, $\varphi = \pi/2$, and the laser beam is continuous ($\sigma'_y \to \infty$) it requires special treatment. Moreover, it is of importance to account for laser beam displacement off the $x, 0, y$ plane.

We suggest the laser density distribution in a bi-Gaussian form:

$$n_{\rm ph} = \frac{N_\parallel}{2\pi\sigma'_x\sigma'_z}\exp\left(-\frac{x'^2}{2\sigma'^2_x} - \frac{z'^2}{2\sigma'^2_z}\right), \qquad (17)$$

where $N_\parallel$ is the longitudinal density (number of photons per unit length of the laser wire).

Making use of (4), for $\varphi = \pi/2$ (integration is made for space dimensions and time over infinite intervals) we get

$$Y(\varphi = \pi/2) = D(z_0)\frac{\sigma N_\parallel N_e}{\sqrt{2\pi(\sigma_z^2 + \sigma'^2_z)}}. \qquad (18)$$

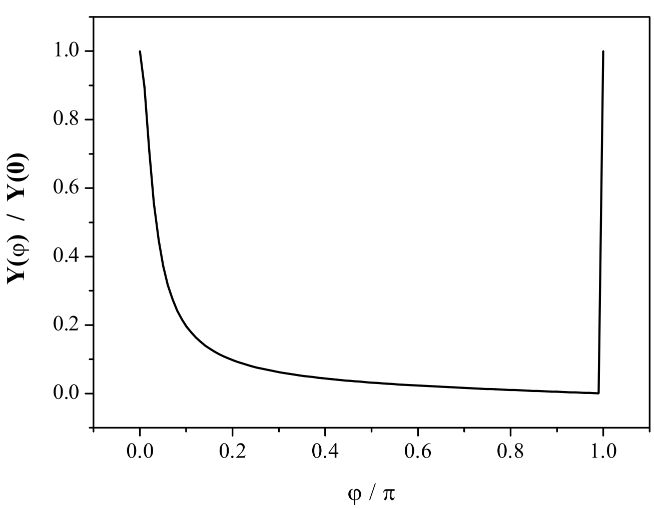

FIG. 3. Total yield versus collision angle $Y(0) = 280$.

TABLE I. Parameters of colliding bunches.

Electron bunch	
Length (σ_y)	10 mm
Radius ($\sigma_x = \sigma_z$)	0.1 mm
Electron energy (γ)	88
Bunch charge	1 nC
Bunch population (N_e)	6.24×10^9
Laser pulse	
Length (σ'_y)	1 mm
Radius ($\sigma'_x = \sigma'_z$)	50 μm
Photon energy ($E_{\rm ph}$)	1.1652 eV
Energy of pulse	1 mJ
Photon population in pulse ($N_{\rm ph}$)	5.36×10^{15}

Here the displacement of the laser wire by z_0 along the z axis results in factor $D(z_0)$:

$$D(z_0) = \exp\left(-\frac{z_0^2}{2(\sigma_z^2 + \sigma'^2_z)}\right).$$

Equation (18) is identical to that presented in [11].

III. DURATION OF X-RAY PULSE

A. Method of evaluation

As was shown above, the total yield of x-ray quanta scattered at the collision of the electron bunch with the laser pulse is determined by Eq. (6). The (dummy) variable of integration T, contained in this expression, denotes local time through interaction space. The "retarded time" τ (seen by the observer) depends on a distance R from the interaction center to the observer and on an angular position of the observer with respect to, say, bunch trajectory.

Since in the vast majority of practical schemes backscattering is employed, i.e., x-ray quanta registered or utilized along the electron bunch orbit where their intensity and energy is maximal, we will consider this particular case $\psi = 0$ in Fig. 1.

In order to obtain the x-ray flux at the observation point (the observer location denoted "D" in Fig. 1), let us shift time in (6) by $R = \tau - T$ required by a quantum to come to the observer (τ is the observer's time). Then we get

$$Y(\tau) = \mathcal{G}\iiint \exp\left[-\frac{\mathcal{F}(x, y, z, \tau - R)}{2}\right]{\rm d}x{\rm d}y{\rm d}z. \qquad (19)$$

Explicit dependence of R on spatial coordinates in an approximating case when distance to the observer r is much greater then the size of the radiating region reads $R \approx r - x\sin\psi - y\cos\psi$, with ψ being the scattering angle of x-ray quanta.

Quadratic form $\mathcal{F}(x, y, z, \tau - R)$ looks similar to (7), with elements

$$a_{11} = \frac{1}{\sigma_x^2} + \frac{\cos^2\varphi}{\sigma'^2_x} + \frac{(\sin\varphi - \sin\psi)^2}{\sigma'^2_y} + \frac{\beta^2\sin^2\psi}{\sigma_y^2},$$

$$a_{12} = \left(\frac{1}{\sigma_x'^2} - \frac{1}{\sigma_y'^2}\right)\cos\varphi\sin\varphi + \left(\frac{\beta^2}{\sigma_y^2} + \frac{1}{\sigma_y'^2}\right)\cos\psi\sin\psi - \frac{\beta\sin\psi}{\sigma_y^2} + \frac{\sin(\psi - \varphi)}{\sigma_y'^2},$$

$$a_{22} = \frac{(1 - \beta\cos\psi)^2}{\sigma_y^2} + \frac{\sin^2\varphi}{\sigma_x'^2} + \frac{(\cos\varphi + \cos\psi)^2}{\sigma_y'^2},$$

$$a_{14} = \left(\frac{\beta^2}{\sigma_y^2} + \frac{1}{\sigma_y'^2}\right)\sin\psi - \frac{\sin\varphi}{\sigma_y'^2},$$

$$a_{24} = \left(\frac{\beta^2}{\sigma_y^2} + \frac{1}{\sigma_y'^2}\right)\cos\psi + \frac{\cos\varphi}{\sigma_y'^2} - \frac{\beta}{\sigma_y^2},$$

$$a_{33} = \frac{\sigma_z'^2 + \sigma_z^2}{\sigma_z^2\sigma_z'^2},$$

$$a_{44} = \frac{\beta^2}{\sigma_y^2} + \frac{1}{\sigma_y'^2}.$$

Making use of variables

$$\eta_1 = x + \frac{a_{12}}{a_{11}}y + \frac{a_{14}}{a_{11}}(\tau - r),$$

$$\eta_2 = y + \frac{a_{11}a_{24} - a_{12}a_{14}}{a_{11}a_{22} - a_{12}^2}(\tau - r),$$

$$\eta_3 = z,$$

$$\eta_4 = \tau - r,$$

the form $\mathcal{F}$ becomes diagonal similar to (8), where the elements $\mathcal{A}_{ij}$ are expressed via a_{ij} in the way of (9).

After a reduction of the quadratic form to the diagonal kind and successive integration, a general dependence of the x-ray intensity upon an observer's time (19) reads

$$Y(\tau) = \frac{\Delta}{2\pi\sigma_\tau}\exp\left[-\frac{(\tau - r)^2}{2\sigma_\tau^2}\right], \tag{20}$$

with

$$\Delta = \frac{2\pi\mathcal{G}}{\sqrt{\mathcal{A}_{11}\mathcal{A}_{22}\mathcal{A}_{33}\mathcal{A}_{44}}},$$

and $\sigma_\tau^2 = 1/\mathcal{A}_{44}$.

Thus, a general expression for temporal dependence of the x-ray intensity is derived. The expression is derived for arbitrary collision angles φ and also arbitrary observation (scattering) angle ψ. As was anticipated, the temporal shape of an x-ray pulse has a Gaussian form with the dispersion σ_τ (equal to rms duration of the x-ray bunch).

B. Duration of backscattered x-ray pulse $\psi=0$

The vast majority of applications employ the backscattering scheme, with $\psi = 0$. It simply stems from the fact that along the electron bunch trajectory the intensity of scattered quanta and their energy are maximal.

Let us consider the case of backscattering, $\psi = 0$ with the arbitrary interaction angle φ. In this case, the expression for σ_τ should be simplified and read

$$\sigma_\tau^2 = \frac{\sigma_y^2[\sigma_y'^2\sin^2\varphi + (\sigma_x^2 + \sigma_x'^2)(1 + \cos\varphi)^2] + (1 - \beta)^2[\sigma_x'^2\sigma_y'^2 + \sigma_x^2(\sigma_y'^2\cos^2\varphi + \sigma_x'^2\sin^2\varphi)]}{\sigma_x^2(\beta + \cos\varphi)^2 + \sigma_x'^2(1 + \beta\cos\varphi)^2 + (\sigma_y^2 + \beta\sigma_y'^2)\sin^2\varphi}. \tag{21}$$

Duration at specific collision angles

As can be seen, for the head-on collision of ultrarelativistic electrons ($\varphi = 0$ and $\gamma \gg 1$), the time duration (21) casts into a known form [12],

$$\sigma_\tau^2 = \frac{4\sigma_y^2 + \sigma_y'^2(1 - \beta)^2}{(1 + \beta)^2} \approx \sigma_y^2 + \frac{\sigma_y'^2}{16\gamma^4}, \tag{22}$$

i.e., length of the x-ray bunch is insignificantly longer than the electron bunch length.

For collision at the right angle $\varphi = \pi/2$,

$$\sigma_\tau^2 = \frac{\sigma_y^2(\sigma_x^2 + \sigma_x'^2 + \sigma_y'^2) + \sigma_x'^2(\sigma_x^2 + \sigma_y'^2)(1 - \beta)^2}{\sigma_y^2 + \sigma_x'^2 + \beta^2(\sigma_y'^2 + \sigma_x^2)} \approx \sigma_y^2\rho\left[1 + \frac{\rho\nu}{\gamma^2} + \frac{\nu^2\rho^2}{\gamma^4}\left(1 + \frac{1}{4\nu\rho^2}\frac{\sigma_x'^2}{\sigma_y^2}\right)\right], \tag{23}$$

with $\rho = (\sigma_y'^2 + \sigma_x^2 + \sigma_x'^2)/(\sigma_y^2 + \sigma_y'^2 + \sigma_x^2 + \sigma_x'^2)$, $\nu = (\sigma_y'^2 + \sigma_x^2)/(\sigma_y'^2 + \sigma_x^2 + \sigma_x'^2)$.

At $\gamma \to \infty$, Eq. (23) can be reduced to the known (see [10,13]) equation

$$\sigma_\tau^2 = \frac{\sigma_y^2(\sigma_y'^2 + \sigma_x^2 + \sigma_x'^2)}{\sigma_y^2 + \sigma_y'^2 + \sigma_x^2 + \sigma_x'^2}. \tag{24}$$

As follows from (23) and (24), for sufficiently long electron bunches $\sigma_y \gg \sigma_y'$, the duration is mainly determined by the length of electron bunch and the transverse sizes of both bunches: $\sigma_\tau^2 \approx \sigma_y'^2 + \sigma_x^2 + \sigma_x'^2$.

For the long photon bunches $\sigma_y' \gg \sigma_y$, or in the case of laser wires ($\sigma_y' \to \infty$, $\rho \to 1$, $\nu \to 1$), it is determined by the length of electron bunch and the width (radius) of photon pulse.

It is obvious that the duration of an x-ray pulse generated at the back-on collision ($\varphi = \pi$) is equal to the duration of the laser pulse σ_y'. At collision angles close to π (and $\gamma \gg 1$), the x-ray pulse duration is

$$\sigma_\tau^2 \approx \sigma_y'^2\left[1 - \frac{\sigma_y'^2 \mu_1 \mu_2}{\sigma_x^2 + \sigma_x'^2}(\pi - \varphi)^2\right], \qquad (25)$$

with $\mu_1 = 2\gamma^2 - \sigma_x^2/\sigma_y'^2$, $\mu_2 = 2\gamma^2 + \sigma_x'^2/\sigma_y'^2$. (This expression differs from the one presented in [14].)

Parameter μ_1 determines the slope of the curve $\sigma_\tau = \sigma_\tau(\varphi)$ while φ approaches π. If $\mu_1 > 0$, i.e., $\sigma_x < \sqrt{2}\gamma\sigma_y'$, the duration increases at $\varphi \to \pi$. If $\mu_1 < 0$, the duration decreases.

For the case when the condition $\sigma_y \gg \sigma_y' \gg \sigma_x \sim \sigma_x'$ holds, a degree of smallness of the collision angle provided the relation (25) is valid is determined by nonequality: $\pi - \varphi \ll \sigma_x/\gamma^2\sigma_y'$. Thus, when φ approximates to π, a sharp rise in duration occurs up to maximum σ_y'.

The dependence of the x-ray pulse duration upon the bunches dimensions can be demonstrated considering the duration of x-ray pulse generated by a single electron ($\sigma_x = \sigma_y = 0$),

$$\sigma_\tau^2 = \frac{\sigma_y'^2 \sigma_x'^2 (1-\beta)^2}{\sigma_x'^2(1+\beta\cos\varphi)^2 + \beta^2\sigma_y'^2\sin^2\varphi}, \qquad (26)$$

and by a single photon ($\sigma_x' = \sigma_y' = 0$),

$$\sigma_\tau^2 = \frac{\sigma_y^2\sigma_x^2(1+\cos\varphi)^2}{\sigma_x^2(\beta+\cos\varphi)^2 + \sigma_y^2\sin^2\varphi}. \qquad (27)$$

The "single particle" pulse durations as functions of the collision angle are presented in Fig. 4.

As can be seen from Eq. (26) and the curve in Fig. 4, the main impact of the laser pulse dimensions is observed at large collision angles, $\pi - \varphi \lesssim 1/\gamma^2$. The minimal duration ($\sigma_\tau = \sigma_y'/4\gamma^2$) is at the head-on collision; the maximal ($\sigma_\tau = \sigma_y'$) at the back-on.

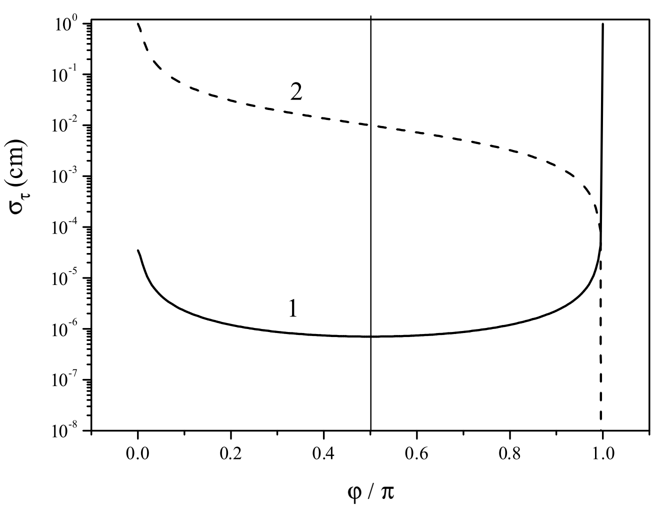

FIG. 4. Single electron x-ray pulse duration (1) and single photon (2) at $\sigma_y = \sigma_y' = 1$ cm, $\sigma_{x,z} = \sigma_{x,z}' = 100$ μm, $\gamma = 88$.

To the contrary, the electron bunch sizes impact largely at small collision angles (close to head-on), as follows from Eq. (27) and Fig. 4.

Thus, a minimal duration of x-ray pulse sufficiently depends upon electron energy γ and occurs near the back-on collision angle $\pi - \varphi \ll 1$.

For the practical case of the parameters of bunches from Table I, $\sigma_y \gg \sigma_y' \gg \sigma_x \sim \sigma_x'$, the x-ray pulse duration is plotted in Fig. 5.

As can be seen from the figure, the pulse duration at the ends of the interval $0 \le \varphi \le \pi$ is determined by the length of electron bunch and the laser pulse, respectively. The minimal duration occurs close to the back-on case: $\pi - \varphi \sim \gamma^{-2}$. Within the interval $\pi/5 \le \varphi < \pi$, the pulse duration practically does not vary.

C. Laser wire monitor

The duration of the x-ray pulse for the laser monitor can be derived by following the general approach developed in Sec. III A. The photon density distribution function of (17) should be provided; in the expressions for elements a_{ij} set $\psi = 0$ and $\sigma_y' \to \infty$.

Much more easily the duration of x-ray pulse generated by the bunch crossing the laser wire can be derived from (21), which allows limit transition $\sigma_y' \to \infty$. Utilizing both methods we get the same formula:

$$\sigma_\tau^2 = \frac{\sigma_y^2\sin^2\varphi + [\sigma_x'^2 + \sigma_x^2\cos^2\varphi](1-\beta)^2}{\beta^2\sin^2\varphi}, \qquad (28)$$

which in the ultrarelativistic limit $\gamma \gg 1$ reduces to

$$\sigma_\tau^2 = \sigma_y^2\left[1 + \frac{1}{\gamma^2} + \left(1 + \frac{\sigma_x'^2 + \sigma_x^2\cos^2\varphi}{4\sigma_y^2\sin^2\varphi}\right)\frac{1}{\gamma^4}\right]. \qquad (29)$$

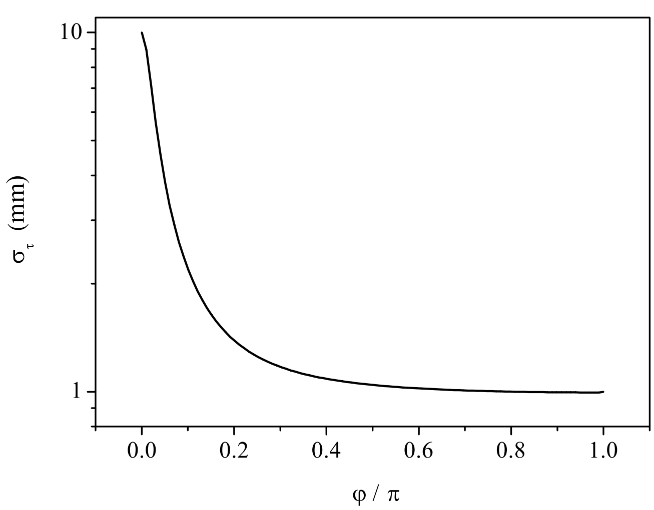

FIG. 5. Backscattered x-ray quanta pulse duration.

IV. SUMMARY AND DISCUSSION

The model of colliding electron bunches with a laser pulse has been considered. The density distribution within the bunches is suggested to be tri-Gaussian, with the bunches dimensions given. The bunches cross each other at arbitrary angles.

The closed analytical expressions for the yield of secondary x-ray quanta and for the x-ray pulse duration observed along the electron bunch trajectory were derived.

As was shown, the maximum yield produces the head-on collision. Decrease in the yield with enlarging the collision angle depends on a ratio of transverse sizes to longitudinal: the smaller the ratio the bigger the decrease.

A procedure of deriving a general analytical formula for the x-ray pulse duration was developed. The closed analytical expression for pulse duration is derived for the practical case of observation along the electron bunch trajectory, $\psi = 0$. We studied the response of the pulse duration for a change of bunch dimensions and energy of electrons for the limiting cases of head-on collision and side-on collision (bunches cross at a right angle). As was found, the latter case, $\varphi = \pi/2$, is not special (does not produce the shortest x-ray pulse as would be implied). Moreover, the pulse duration remains almost the same within a wide range of colliding angles.

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